B. Sc - II , Paper - III , Gr - B

Maxwell's Equations Part - I :

Divergence and Curl of $\vec{E}$ and $\vec{B}$ (electric and magnetic fields) before Maxwell fixed the Ampere's law can be written as

Represents

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \text{—— (i)} \rightarrow \text{Gauss's law}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{—— (ii)} \rightarrow \text{No name}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{—— (iii)} \rightarrow \text{Faraday's law}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad \text{—— (iv)} \rightarrow \text{Ampere's law}$$

where, $\rho \rightarrow$ electric charge density, $\vec{J} \rightarrow$ current density

Since we know that divergence of a curl is zero.

Now taking divergence of Eq (iii) both sides

$$\nabla \cdot (\nabla \times \vec{E}) = -\nabla \cdot \left(\frac{\partial \vec{B}}{\partial t} \right)$$

$$= -\frac{\partial}{\partial t} (\nabla \cdot \vec{B})$$

LHS = 0 ~~RHS = 0~~ according to the properties of divergence and curl.

RHS is zero from eq. (ii).

But if we take divergence of Eq (iv) on both sides we obtain

$$\nabla \cdot (\nabla \times \vec{B}) = \mu_0 (\nabla \cdot \vec{J})$$

LHS = 0 → from the definition

But RHS will be zero if $\vec{J}$ is steady state current. This implies that Ampere's law defined by eq. (iv) will not work if we work ~~with~~ beyond magnetostatics. There is theoretical fault in eq. (iv) for general situation. which Maxwell fixed by adding extra term on rhs of eq. (iv) using some theoretical argument. In the next class we will discuss how Maxwell fixed Ampere's law.